DERIVATION OF THE THERMAL RESISTANCE OF THE CONTACT BETWEEN SYSTEMS WITH CORRUGATED SURFACES

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Contact thermal resistance is considered for joints with corrugated surfaces. Formulas are derived that are confirmed by experiment.

There are [1-3] fairly many papers on heat transfer in the contact zones of solids; but these in the main deal with joints with rough flat surfaces.

On the other hand, most industrial finishes have [4, 5] deviations from planarity as periodic ridges and depressions with separations considerably larger than the microroughness of the surface; such surfaces may be called corrugated.

When corrugated surfaces are in contact, there are discrete points of contact between the vertices of projections on the ridges; the deformation and hence the production of the actual area of contact and the gap are in this case different from those for planar surfaces [4, 5].

The mode of contact heat transfer is [2] completely determined by the actual area of contact and the size of the gap between the surfaces.

Here we consider the effects of corrugation on such a contact and the possible features of the heat transfer between such surfaces.

The thermal resistance of a contact may [2] be expressed as two resistances in parallel:

$$\frac{1}{R_c} = \frac{1}{R_м} + \frac{1}{R_{cl}}. \qquad (1)$$

The following formula defines [6] the thermal resistance via spots of actual contact on account of convergence of heat-flux lines to the points:

$$R_м = \frac{\varphi S_0}{2a\bar{\lambda}_м n}. \qquad (2)$$

The coefficient φ for convergence of these lines can be expressed via the relative area of actual contact [7]:

$$\varphi = 1 - 1.41\eta_l^{1/2} + 0.3\eta_l^{3/2}. \qquad (3)$$

If we assume [8] that $a = 3\cdot10^{-5}$ m and $n = S_m/\pi a^2$, (2) may be put as

$$R_м = \frac{\varphi}{2.12\bar{\lambda}_м\eta_1}\cdot10^{-4}. \qquad (4)$$

The relative contact area appears in (4) and is found from an equation [5] for the most common elastic deformation of the roughness:

$$\eta = \left(\frac{\alpha^{\omega/\nu}\, b^{\omega/\nu}\, r^{\omega} q_c}{h_{max}^{\omega} KB}\right)^{\frac{\nu}{\nu+\omega}}. \qquad (5)$$

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TABLE 1. Characteristics of Specimens and Conditions

Pair No.	Material	Finish	Treatment	Mean roughness height, μ	Contact temp., °K	$q_{av}, 10^{-3}$ w/m²	Notes
1.	1Kh18N9T / 1Kh18N9T	V/∇8"b" — ∇9"b"	milling / grinding	14/2,2 —1,04	443	69,8	One surface with regular spherical waves
2.	1Kh18N9T / 1Kh18N9T	∇5 — ∇9"c"	milling / grinding	13,5—0,9	443	66,8	Rough flat surfaces
3.	1Kh18N9T / 1Kh18N9T	III/∇8"b" — ∇9"b"	milling / grinding	3,2/2,4 —1,2	445	69,6	One surface with regular cylindrical waves
4.	1Kh18N9T / 1Kh18N9T	∇7"c"— ∇9"b"	milling / grinding	3,4—1,04	445	67,2	Rough flat surfaces
5.	D16T / steel 45	VII/∇6"b" — ∇6"b"	milling / grinding	46/7,9 —6,1	383	64,9	One surface with regular cylindrical waves
6.	D16T / steel 45	∇3 — ∇6"b"	milling / grinding	44,6—6,1	383	57,2	Rough flat surfaces
7.	steel 45	V/∇7"a" —∇8"c"	milling / grinding	12/6 —1,9	423	103	One surface with irregular spherical waves
8.	1Kh18N9T / 1Kh18N9T	V/∇8"b" — ∇9"b"	milling / grinding	14/2,2 —1,04	440	—	Vacuum between surface.

However, to use (5) we need to measure a large number of parameters for the surfaces. To facilitate the design calculations, we have examined numerous shapes of surfaces together with the physical and mechanical characteristics for finish classes from 3 to 10, after the surfaces have previously been loaded to $P/E = 5 \cdot 10^{-6}$-$5 \cdot 10^{-4}$.

Then the values may be processed via (5) to give

$$\eta_1 = \left(\frac{q_c}{E} A\right)^{0,8}, \tag{6}$$

where Fig. 1 shows coefficient A as a function of $h_{av_1} + h_{av_2}$.

From (4) and (6) we have

$$R_M = \frac{\varphi}{2.12\bar{\lambda}_M \left(\frac{q_c}{E} A\right)^{0,8} \eta_2} \cdot 10^{-4}. \tag{7}$$

One can use (7) with known values of the contact area S_c in η_2; the contour area of contact is given by Hertz's formula [9]. This can be simplified if we assume [10] that the areas of contact for spherical and ellipsoidal corrugations are given by the same relationship, and also that $1-\mu^2 = 0.9$:

for contact of two surfaces of spherical or ellipsoidal corrugations

$$S_{c\,sp-sp} = 2.38 \left[\frac{r_{1w} r_{2w} N n_w^{1/2}}{(r_{1w} + r_{2w})E}\right]^{2/3}, \tag{8}$$

for contact between rough flat and corrugated surfaces

$$S_{c\,sp\cdot fl-r} = 2.38 \left(\frac{r_w N n_w}{E}\right)^{2/3}. \tag{9}$$

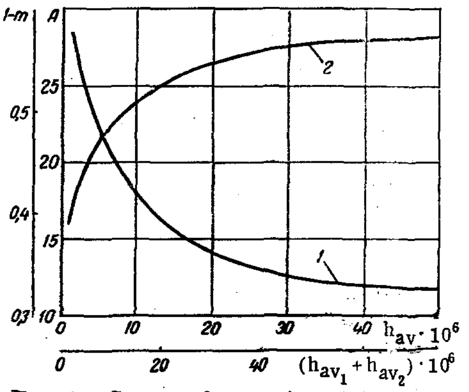

Fig.1. Curves for 1) $A = f(h_{av_1} + h_{av_2})$ and 2) $1-m = f(h_{av})$.

The number of contact spots on the waves is taken as $n_w = 3$ for small loads and $n_w = S_0/L_{el} L_p$ for high loads. If the surfaces have cylindrical corrugations, the contour area of contact is defined by the following formulas:

contact between two cylindrical corrugated surfaces

$$S_{c.c-c} = 1.52\left(\frac{r_{1w} r_{2w}}{r_{1w}+r_{2w}}\cdot\frac{NL}{E}\right)^{1/2}, \tag{10}$$

contact of a flat rough surface with a corrugated one

$$S_{c.c\cdot fl-r} = 1{,}52\left(\frac{r_w NL}{E}\right)^{1/2}. \tag{11}$$

The following formula defines the thermal resistance when the heat is transferred via a contact medium of low thermal conductivity:

$$R_{cl} = \delta_{eq}/\lambda_c. \tag{12}$$

The equivalent thickness δ_{eq} for the layer between the contacting surfaces is found as follows:

for contact between corrugated surfaces

$$\delta_{eq} = (H_{av_1} + H_{av_2})(1-K)(1-\varepsilon), \tag{13}$$

for contact between a corrugated and a rough flat surface

$$\delta_{eq} = [H_{av}(1-K) + h_{av}(1-m)](1-\varepsilon). \tag{14}$$

The difference $1-m$ appearing in (13) and (14) is shown in Fig.1 as a function of h_{av}; this result was obtained by processing numerous profile curves for specimens with surface finishes of classes from 3 to 10 for metals having $E > 7\cdot10^{10}$ N/m^2.

We derived the numerical value of the shape filling factor for the corrugations by processing published profiles [11] and also patterns recorded from corrugated surfaces; for most surfaces the result was in the range 0.45-0.5.

The relative approach $\varepsilon = c/h_{max}$ is [5] given by

$$\varepsilon = \left(\frac{r^{\omega} q_c}{\alpha b K B h_{max}^{\omega}}\right)^{\frac{1}{\nu+\omega}}. \tag{15}$$

Use of (15) involves the same difficulties as for η_1; it is much easier to use relationships derived from (15) by processing data on the geometry and properties of surfaces with various forms of mechanical working:

shaping, turning, and milling

$$\varepsilon = \left(\frac{0{,}1 q_c}{HB}\right)^{0{,}28}, \tag{16}$$

grinding

$$\varepsilon = \left(\frac{0{,}0125 q_c}{HB}\right)^{0.185}, \tag{17}$$

polishing and honing

$$\varepsilon = \left(\frac{0.0064 q_c}{HB}\right)^{0.185}. \tag{18}$$

We substitute (7) and (12)-(14) into (1) to get the following equation for the thermal resistance of joints with corrugated surfaces:

contact between corrugated surfaces

$$\frac{1}{R_c} = 2.12\,\bar{\lambda}_M\left(\frac{q_c}{E}A\right)^{0,8}\frac{\eta_2}{\varphi} + \frac{\lambda_c}{(H_{av} + H_{av})(1-K)(1-\varepsilon)}, \tag{19}$$

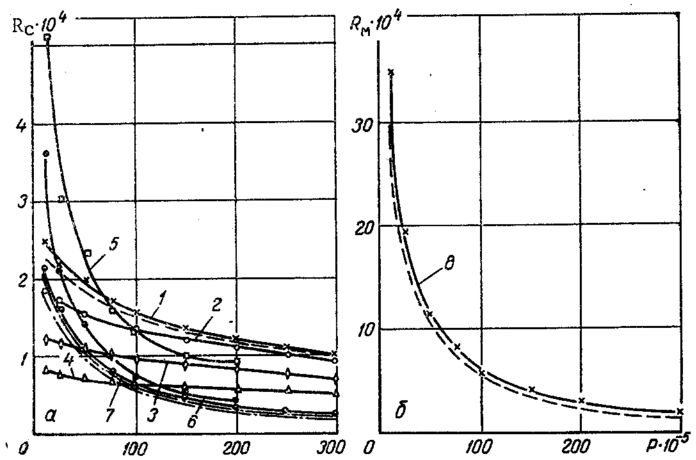

Fig.2. Thermal resistance of contact, $m^2 \cdot deg/W$, as a function of contact pressure (N/m^2): a) wavy surfaces: 1, 3, 5) regular; 7) irregular; 2, 4, 6) no waves, air in space (the numbers of the pairs are as in Table 1), b: 9) waves present, space evacuated: broken line) calculation; upper dot-and-dash line) contour area of contact defined by (9); lower dot-and-dash line) contour area of contact measured by the color method.

contact between corrugated and rough flat surfaces

$$\frac{1}{R_c} = 2.12\,\bar{\lambda}_м \left(\frac{q_e}{E} A\right)^{0,8} \frac{\eta_2}{\varphi} + \frac{\lambda_c}{[H_{av}(1-K)+h_{av}(1-m)](1-\varepsilon)} \tag{20}$$

Numerous assumptions are involved in deriving (19) and (20), and the justification for these can only be experimental.

We used a system of rod type [12] to measure the thermal resistance of contacts between metal surfaces in the presence and absence of corrugations. Table 1 gives the characteristics of the specimens. To eliminate effects from the individual projections and microroughness, the specimens were first compressed to $P/E = (6\text{-}6.5) \cdot 10^{-4}$ for up to 20 min.

We used the instrument termed Kalibr BÉI to measure the surface profiles against the flat reference plane in the instrument; the corrugation class was determined from the upper limit to the wave height in accordance with the All-Union State Standard 2789-59.

Figure 2a gives the experimental and calculated results as $R_c = f(P)$ for corrugated and flat surfaces; we compared pairs in one case having a rough flat surface and a corrugated surface, and in the other having the corrugated surface replaced by a rough flat one whose nonuniformities had a height equal to the height of the corrugations.

It is clear that (19) and (20) describe the thermal resistance as a function of pressure with accuracy sufficient for design calculations.

The experimental tests also revealed numerous interesting features of the heat transfer through joints between corrugated surfaces; corrugation on even one of the two surfaces raised the thermal resistance relative to that between rough flat surfaces, and the rise in R_c was largest in the range of initial loads (up to $5 \cdot 10^{-6}$ N/m^2).

The corrugation height had the main influence on the thermal resistance. Figure 2 shows that the resistance was increased by more than an order of magnitude when the pair consisted of steel 1Kh18N9T with parameters $L = 25 \cdot 10^{-4}$ m and $H = 0.14 \cdot 10^{-4}$ m, the comparison being with the corrugated surface in which $L = 50 \cdot 10^{-4}$ m and $H = 0.032 \cdot 10^{-4}$ m.

The curves of Fig.2 are shown also as $R_M = f(P)$. It is clear that the actual area of contact controls the overall thermal resistance when the surfaces are corrugated.

It is common industrial practice to use surfaces having irregular corrugation. Figure 2a shows results for the case where S_c was measured by the die method and where it was calculated from (9); the two sets of curves agree satisfactorily. The following suggestions may be made on the basis of the above results. If there is thermal contact between corrugated surfaces up to class III, one can assume that R_c will not exceed the value for rough flat surfaces with nonuniformities of height equal to the height of the corrugations; this assumption is the more justified because R_c does not exceed 10^{-4} $m^2 \cdot deg/W$ for many common systems with heat fluxes of $q_{av} = 10\text{–}60 \cdot 10^3$ W/m^2.

NOTATION

R_c	is the total thermal resistance of contact, $m^2 \cdot deg/W$;
R_M, R_{cl}	are the thermal resistance of real contact and of contactless region, $m^2 \cdot deg/W$;
φ	is the coefficient of contraction of heat flux lines to spots of real contact;
S_m, S_c, S_o	are the real, contour and nominal areas of contact surfaces, m^2;
a	is the mean radius of contact spot, m;
$\bar{\lambda}_M$	is the reduced thermal conductivity of contact (1 and 2) materials, $W/m \cdot deg$;
λ_c	is the thermal conductivity of contact medium, $W/m \cdot deg$;
n	is the number of contact spots of microroughnesses at nominal contact surface;
α	is the area ratio;
b, ν	are the parameters of support curve of surface;
r	is the radius of roughness, m;
q_c	is the contour pressure, N/m^2;
N	is the normal load, N;
ω	is the coefficient depending on deformation mechanism;
B	is the coefficient characterizing properties;
K	is the coefficient depending on ν and ω;
h_{max}, h_{av}	are the maximum and mean height of microroughness protrusions, m;
P	is the specific normal load to contact surface, N/m^2;
E	is the Young's modulus, N/m^2;
r_w	is the wave radius, m;
n_w	is the numbers of wave contact spots at nominal surface;
L_{el}, L_p	are the longitudinal and transverse wave pitch, m;
δ_{eq}	is the equivalent thickness of intercontact laminar, m;
H_{av}	is the mean height of waves, m;
ε	is the relative approach of surfaces under load;
c	is the approach of surfaces under load;
HB	is the Brinell hardness, N/m^2;
μ	is the Poisson's ratio;
η_2	are the relative contact surfaces.

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